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VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

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- Statistics basics: data analysis, central tendency, Dispersion, QD, Mean deviation, variance, SD
- Probability
- Theoretical distribution – 3 types

1. *Binomial*
2. *Poisson*
3. *Normal*

Chap 16 : Discussion flow

- Probability definition, concepts
- Conditional probability
- Expectation
- Variables – discrete & continuous
- Problems & solution – Ex 16 set A,B,C
- CA MTP 2020

16.1 INTRODUCTION

- 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against'
- probability has developed itself into a full-fledged subject and become an integral part of statistics.

Probability :

At the Foundation level the concept of Probability is used in accounting and finance to understand the likelihood of occurrence or non-occurrence of a variable. It helps in developing financial forecasting in which you need to develop expertise at an advanced stage of chartered accountancy course. Here in this capsule an attempt is made for solving and understanding the concepts of probability.

Concept & Definition

The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability. In recent time, probability has developed itself into a full-fledged subject and become an integral part of statistics.

Random Experiment: An experiment is defined to be random if the results of the experiment depend on chance only. For example if a coin is tossed, then we get two outcomes—Head (H) and Tail (T). It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once. Thus, tossing a coin is an example of a random experiment. Similarly, rolling a dice (or any number of dice), drawing items from a box containing both defective and non—defective items, drawing cards from a pack of well shuffled fifty—two cards etc. are all random experiments.

16.2 RANDOM EXPERIMENT

- **Experiment:** An **experiment may be described as a performance that produces certain results or EVENTS**
- For example **if a coin is tossed**, then we get two outcomes—Head (H) and Tail (T).
- It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once.
- Thus, tossing a coin is an example of a random experiment

Events= result / Outcome of random experiment

- **Events:** The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types:
- Simple or Elementary,
- Composite or Compound.
- An event is known to be simple if it cannot be decomposed into further events. Tossing a coin once provides us two simple events namely Head and Tail. On the other hand, a composite event is one that can be decomposed into two or more events. Getting a head when a coin is tossed twice is an example of composite event as it can be split into the events HT and TH which are both elementary events.

Important definitions- **ME, EE, EL Events**

- **Mutually Exclusive Events or Incompatible Events:** A set of events A1, A2, A3, …… is known to be **mutually exclusive if not more than one of them can occur** simultaneously. Thus occurrence of one such event implies the non-occurrence of the other events of the set. Once a coin is tossed, we get two mutually exclusive events Head and Tail.
- **Exhaustive Events:** The events A1, A2, A3, ………… are known to form an exhaustive set **if one of these events must necessarily occur**. As an example, the two events Head and Tail, when a coin is tossed once, are exhaustive
- **Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:**
- The two events **Head and Tail when a coin is tossed** is an example of a pair of equally likely events because there is no reason to assume that Head (or Tail) would occur more frequently as compared to Tail (or Head).

16.3 CLASSICAL DEFINITION OF PROBABILITY OR A PRIOR DEFINITION

- For this definition of probability, we are indebted to **Bernoulli and Laplace.**
- This definition is also termed as **a priori definition** because probability of the event A is defined on the basis of prior knowledge.

$$P(A) = \frac{n_A}{n} = \frac{\text{No. of equally likely events favourable to A}}{\text{Total no. of equally likely events}}$$

$$P(A) = \frac{m_A}{m} = \frac{\text{No. of mutually exclusive, exhaustive and equally likely events favourable to A}}{\text{Total no.of mutually exclusive, exhaustive andequally likely events}}$$

However, if instead of considering all elementary events, we focus our attention to only those composite events, which are mutually exclusive, exhaustive and equally likely and if $m(\leq n)$ denotes such events and is furthermore $m_A(\leq n_A)$ denotes the no. of mutually exclusive, exhaustive and equally likely events favourable to A, then we have

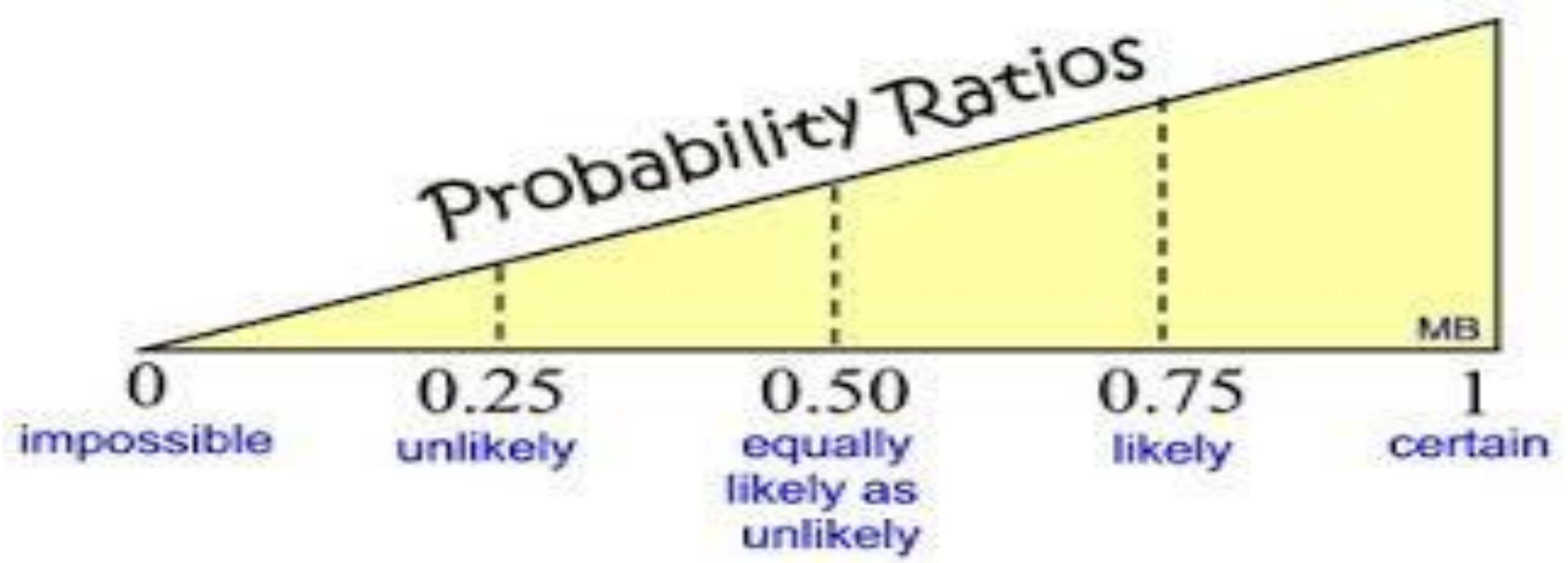
$$P(A) = \frac{m_A}{m} = \frac{\text{"Number of mutually exclusive, exhaustive and equally likely events favourable to A"}}{\text{"Total Number of mutually exclusive, exhaustive and equally likely events"}}$$

Complementary event

- The probability of an event lies between 0 and 1, both inclusive.
- i.e. 0 <= P(A) <= 1……. (16.3)
- When P(A) = 0, A is known to be an **impossible event** and when P(A) = 1, A is known to be a **sure event.**
- Non-occurrence of event A is denoted by A' or A^{C} or □ and it is known as complimentary event of A. The event A along with its complimentary A' forms a set of mutually exclusive and exhaustive events.
- i.e. **P(A) + P (A') = 1**
- The **ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event A and its inverse ratio is known as odds against the event A.**
- **i.e. odds in favour of A**
- **= mA : (m – mA) ……………… (16.5)**
- **and odds against A = (m – mA) : mA , ……………… (16.6)**

Concept

Non-occurrence of event A is denoted by A' and it is known as complimentary event of A. The event A along with its complimentary A' forms a set of mutually exclusive and exhaustive events. i.e. P(A) + P (A') = 1



ILLUSTRATIONS:

- **Example 16.1:** A coin is tossed three times. What is the probability of getting:
- 2 heads
- at least 2 heads.
- **Solution:** When a coin is tossed three times, first we need enumerate all the elementary events. This can be done using 'Tree diagram' as shown below:
- HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
- Thus the number of elementary events (n) is 8.
- Out of these 8 outcomes, 2 heads occur in three cases namely HHT, HTH and THH., we have
- P (A)= $\underline{3} / 8$ =0.375

Solution

- B) Let B denote occurrence of at least 2 heads i.e. 2 heads or 3 heads. Since 2 heads occur in 3 cases and 3 heads occur in only 1 case, B occurs in 3 + 1 or 4 cases. By the classical definition of probability,
- P(B) = 4/8 = 0.50

Illustration

- **Example 16.2:** A dice is rolled twice. What is the probability of getting a difference of 2 points?
- **Solution:** If an experiment results in p outcomes and if the experiment is repeated q times, then the total number of outcomes is pq.
- In the present case, since a dice results in 6 outcomes and the dice is rolled twice, total no. of outcomes or elementary events is 62 or 36. We assume that the dice is unbiased which ensures that all these 36 elementary events are equally likely.
- Now a difference of 2 points in the uppermost faces of the dice thrown twice can occur in the following cases:

Analysis

1st Throw	2nd Throw	Difference
6	4	2
5	3	2
4	2	2
3	1	2
1	3	2
2	4	2
3	5	2
4	6	2

Thus denoting the event of getting a difference of 2 points by A, we find that the no. of outcomes favourable to A, from the above table, is 8. By classical definition of probability, we get

P(A) = 8 / 36

= 2 / 9

- **16.5 :** A sample space may be defined as a non-empty set containing all the **elementary events of a random experiment as sample points.**
- If a dice is rolled once than the sample space is given by S = {1, 2, 3, 4, 5, 6}.
- Next, if we define the events A, B and C such that A = {x: x is an even no. of points in S}
- B = {x: x is an odd no. of points in S}
- C = {x: x is a multiple of 3 points in S}
- Then, it is quite obvious that
- A = {2, 4, 6}, B = {1, 3, 5} and C = {3, 6}.

- In the above example, we have AUC = {2, 3, 4, 6}
- and AUB = {1, 2, 3, 4, 5, 6}.
- The intersection of two events A and B may be defined as the set containing all the sample points that are common to both the events A and B.
- . we have
- AnB = {x:x□A and x□B }.
- In the above example, A n B = null set
- A n C = {6}

- Two events A and B are mutually exclusive if P (A n B) = 0 or more precisely
- **P (A U B) = P(A) + P(B)**   (16.10)
- Similarly three events A, B and C are mutually exclusive if
- **P (A U B U C) = P(A) + P(B) + P(C)** (16.11)
- Two events A and B are exhaustive if
- **P(A U B) = 1**   (16.12)
- Similarly three events A, B and C are exhaustive if
- **P(A U B U C) = 1**   (16.13)
- Three events A, B and C are equally likely if
- **P(A) = P(B) = P(C)   (16.14)**

- **Example 16.8:** Three events A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of A?
- **Solution:** Since A, B and C are mutually exclusive, we have
- P(A U B U C) = P(A) + P(B) + P(C) =(1)
- Since they are exhaustive, P(A U B U C) =1 (2)
- Since they are also equally likely, P(A) = P(B) = P(C) = K, Say (3)
- Combining equations (1), (2) and (3), we have 1 = K + K + K
- □ K = 1/3
- Thus P(A) = P(B) = P(C) = 1/3 Hence P(A’) = 1 – 1/3 = 2/3

- **16.7 ADDITION THEOREMS OR THEOREMS ON TOTAL PROBABILITY**
- **Theorem 1** For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.
- i.e. P (A U B)
- or **P(A + B) = P(A) + P(B) (16.18)**
- or **P(A or B)whenever A and B are mutually exclusive**
- **Example 16.9:** A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?
- **Solution:** Let A be the event that the number selected would be divisible by 4 and B, the event that the selected number would be divisible by 7. Then AUB denotes the event that the number would be divisible by 4 or 7. Next we note that A = {4, 8, 12, 16, 20, 24} and B = {7, 14, 21}
- whereas S = {1, 2, 3, ... 25}. Since A n B = 0 , the two events A and B are mutually exclusive and as such we have

- P(A U B) = P(A) + P(A)......... (1)
- Since P(A) = n(A) /n(S) = 6 / 25
- and P(B) = = n(B)/ n(S) = 3 / 25
- Thus from (1), we have
- P(A U B) = 6/ 25 + 3 /25
- = 9 / 25
- Hence the probability that the selected number would be divisible by 4 or 7 is 9/25 or 0.36

- **Example 16.10:** A coin is tossed thrice. What is the probability of getting 2 or more heads?
- **Solution:** If a coin is tossed three times, then we have the following sample space.
- S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} 2 or more heads imply 2 or 3 heads.
- If A and B denote the events of occurrence of 2 and 3 heads respectively, then we find that A = {HHT, HTH, THH} and B = {HHH}
- P(A)= 3/8
- P(b) = 1/8
- As A and B are mutually exclusive, the probability of getting 2 or more heads is P(A u B) = P(A) + P(B)
- = 3/8 + 1/8 = 0.5

- **Theorem 2** For any K **mutually exclusive events** A1, A2, A3 …, AK the probability that at least one of them occurs is given by the sum of the individual probabilities of the K events.
- i.e. $P(A1 \cup A2 \cup \ldots \cup AK) = P(A1) + P(A2) + \ldots P(AK)$
- **Theorem 3** For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.
- i. e. $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (16.20)}$

- **Important**
- **Example 16.12:** The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?
- **Solution:** Let the event that the applicant is a B. Com. be denoted by B and that he is a CA be denoted by C Then as given,
- P(B) = 0.85, P(C) = 0.30 and P(B n C) = 0.25
- The probability that an applicant is B. Com. or CA is given by P(B U C) = P(B) + P(C) – P(B n C)
- = 0.85 + 0.30 – 0.25= = 0.90
- Expected frequency = N × P (B U C) Expected frequency = 500 × 0.90 = 450

- **Theorem 4** For any three events A, B and C, the probability that at least one of the events occurs is given by
- **P(A U B U C) = P(A) + P(B) + P(C) – P(A n B) – P(A n C) – P(B n C)+ P(A n B n C)..........** (16.23
- **Example 16.14:** **Important**
- There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

- **Solution:** As given P(A) = 0.80, P(B) = 0.60, P(C) = 0.50,
- P(A n B) = 0.46, P(B n C) = 0.32, P(A n C) = 0.48 and P(A n B n C) = 0.26
- The probability that at least one of them survives another 5 years in given by P(A u B u C)
- = P(A) + P(B) + P(C) – P(A n B) – P(A n C) – P(B n C)+ P(A n B nÇ C) ……. (16.23)
- = 0.80 + 0.60 + 0.50 – 0.46 – 0.32 – 0.48 + 0.26
- = 0.90

16.8 CONDITIONAL PROBABILITY AND COMPOUND THEOREM OF PROBABILITY

Compound Probability or Joint Probability

- The probability of **occurrence of two events A and B simultaneously** is known as the **Compound Probability** or **Joint Probability** of the events A and B and is denoted by P(A n B).
- In the first case, if the occurrence of one event, say B, is influenced by the occurrence of another event A, then the **two events A and B are known as dependent events.**
- We use the notation **P(B/A),** to be read **as 'probability of the event B given that the event A has already occurred** (or 'the conditional probability of B given A) to suggest that another event B will happen if and only if the first event A has already happened.

- **P(B/A)= P(AB)/ P(A)**
- Similarly, P(A/B) = **P(AB)/ P(B)**
- **Theorems of Compound Probability**
- **Theorem 5** For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred
- i.e. $P(A \cap B) = P(A) \times P(B/A)$
- **Theorem 6** For any three events A, B and C, the probability that they occur jointly is given by
- $P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B))$

- Important
- Example 16.15: Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?
- **Solution:** Let A denote the event that Rupesh hits the target and B, the event that David hits the target. Then as given,
- P(A) = 5/ 9 , P(B)= 6/11
- and P(A n B) = P(A) × P(B) = 5/9 * 6/11 = 10/33 (as A and B are independent)
- The probability that the target would be hit is given by
- P(A u B) = P(A) + P(B) – P(A n B)
- 5/9 + 6/11 - 10/33
- 79/ 99

- **Example 16.17:** In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?
- **Solution:** Let S and M stand for service holder and male respectively. We are to evaluate P (S / M).
- We note that (S Ç M) represents the event of both service holder and male.
- Thus P(S/M) = P (SM)/ P (M)
- 12/35 / 20/35
- = 0.60

- **Example 16.21:** Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates. What is the probability that Mr. Roy would be selected?
- **Solution:** Denoting the three posts by A, B and C respectively, we have
- P(A)= 1/3 , P(B)= 1/5, P(C) = 1/10
- P(AUBUC)= 1 - { P(AUBUC) ‘’}
- = 1 -- { P(A’’) .P(B’’) . P(C ‘’)} (De Morgans Law)
- 1-- 12/25
- = 13/ 25

- <u>**Important :**</u>
- <u>**Example 16.25:** If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?</u>
- **Solution:** The first ball can be distributed to the 1st box or 2nd box or 3rd box i.e. it can be distributed in 3 ways. Similarly, the second ball also can be distributed in 3 ways. Thus the first tw o balls can be distributed in 32 ways. Proceeding in this way, we find that 8 balls can be distributed to 3 boxes in 3^8 ways which is the total number of elementary events.
- Let A be the event that the first box contains 3 balls which implies that the remaining 5 both must go to the remaining 2 boxes which, as we have already discussed, can be done in 2^5 ways.
- Since 3 balls out of 8 balls can be selected in 8C3 ways, the event can occur in 8C3 ☐ 2^5 ways, thus we have
- P(A) = 8C 3 * 2^5 / 3^8
- = <u>1792</u> / 6561

Recap

- Probability definition, applications
- Probability concepts
- Solved examples – analysis
- Formulae & theorems
- Conditional probability
- CA MTP Oct 2020

Probability of passing = 1

THANK YOU